RAN-2203000205023003

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T. Y. B. Sc. (Sem.-V) Examination October - 2025

Mathematics - Real Analysis-I (Old Course) Paper-XIII MTH-503

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Seat No.:

Student's Signature

Q.1. Answer the following as directed: (Any FIVE) **(10)**

(1) Show that the sequence $\left\{\frac{1}{1+n^2}\right\}_{n=1}^{\infty}$ is monotone.

(2) For $n \in I$, let $t_n = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots+\frac{1}{n!}$, prove that $\{tn\}_{n=1}^{\infty}$ is non-decreasing.

(3) If $S_n = \frac{10^n}{n!}$, find $n \in I$ such that $S_{n+1} < S_n$ whenever $n \geq N$.

(4) Find the limit inferior and the limit superior of the sequence –1,0,3,–1,0,3,–1,0,3,......

(5) Define Cauchy Sequence and give an example of a Cauchy sequence.

(6) Define: Convergence of the series of real numbers $\sum_{n=1}^{\infty} a_n$.

(7) Show that the series $\sum_{n=1}^{\infty} \frac{1+n}{1+2n}$ diverges.

(8) Define Absolute Convergent Series and give one example.

Q.2. Attempt any TWO: **(10)**

(a) If the sequence of real numbers $\{Sn\}_{n=1}^{\infty}$ is convergent, then prove that $\{Sn\}_{n=1}^{\infty}$ is bounded.

(b) If $\{Sn\}_{n=1}^{\infty}$ is a sequence of real numbers, if $c \in R$, and if $\lim_{n\to\infty} S_n = L$, then show that $\lim_{n\to\infty} cs_n = cL$.

(c) For $n \in I$, if $S_n = \frac{1.3.5\ldots(2n-1)}{2.4.6\ldots(2n)}$, then prove that $\{Sn\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} S_n \leq \frac{1}{2}$

Q.3. Attempt any TWO: **(10)**

(a) If $\{Sn\}_{n=1}^{\infty}$ diverges to infinity and if $\{tn\}_{n=1}^{\infty}$ bounded, then prove that $\{s_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

(b) Find the limit inferior and the limit superior of the sequences $\{-n\}_{n=1}^{\infty}$ and $\{\cos(n\pi)\}_{n=1}^{\infty}$

(c) If the sequence of real numbers $\{Sn\}_{n=1}^{\infty}$ converges, then show that $\{Sn\}_{n=1}^{\infty}$ is a Cauchy sequence.

Q.4. Attempt any TWO: **(10)**

(a) If $\sum_{n=1}^{\infty} a_n$ is a convergent series, then show that $\lim\limits_{n \to \infty} a_n = 0$.

(b) Check the convegence the series $1+\frac{1}{2!}+\frac{1}{4!}+\frac{1}{6!}+ \ldots$ and $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)$.

(c) Prove that the series $(1 - 2) - \left(1 - 2^{\frac{1}{2}}\right) + \left(1 - 2^{\frac{1}{3}}\right) - \left(1 - 2^{\frac{1}{4}}\right) + \ldots$ converges.

Q.5. Attempt any TWO: **(10)**

(a) If $\{a_n\}_{n=1}^{\infty}$ is a nonincreasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges, then prove that $\lim\limits_{n \to \infty} na_n = 0$.

(b) For which values of x; does the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^x}$ converge?

(c) Show that the series (i) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ converges; (ii) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.
